Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 20 June 2024

Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced

PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 6 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.

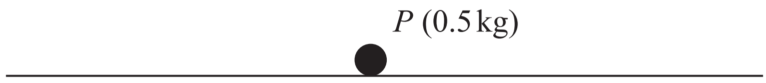


Figure 1

Figure 1 shows a particle P of mass 0.5 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{2}{7}$

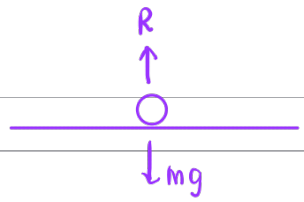
A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

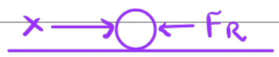
a) when at rest :



Normal, $R = mg$

$= 0.5 \times 9.8 = 4.9\text{ N}$ 1

b) $\mu = \frac{2}{7}$



P is limiting equilibrium : $F_R = \mu \times R$

$\therefore X = F_R$

$= \frac{2}{7} \times 4.9\text{ N}$ 1

$= 1.4$ newton

1

Question 1 continued

(Total for Question 1 is 3 marks)

2.

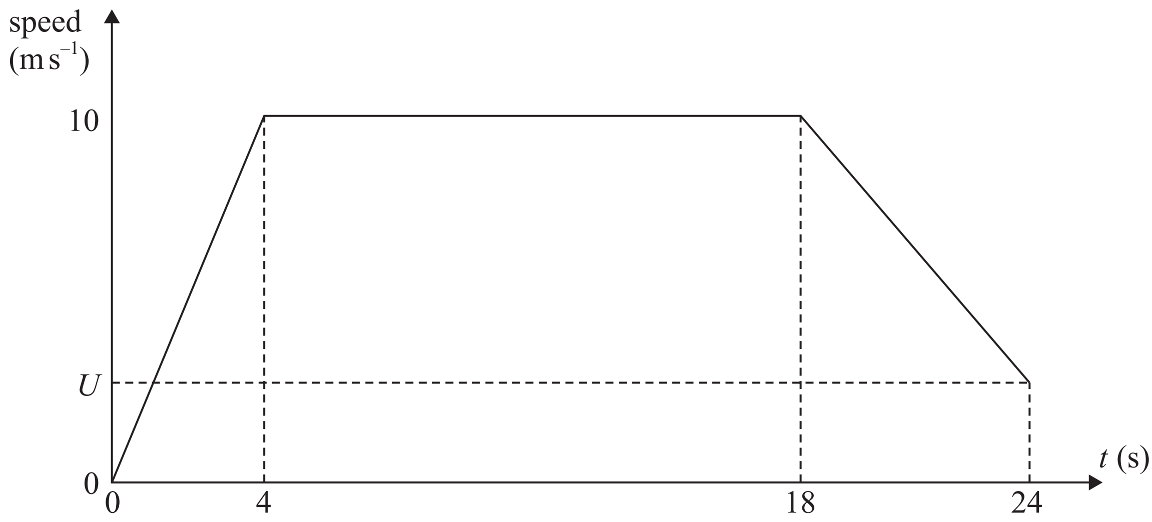


Figure 2

Figure 2 shows a speed-time graph for a model of the motion of an athlete running a **200 m** race in 24 s.

The athlete

- starts from rest at time $t = 0$ and accelerates at a constant rate, reaching a speed of $10\,\mathrm{m\,s^{-1}}$ at $t = 4$
- then moves at a constant speed of $10\,\mathrm{m\,s^{-1}}$ from $t = 4$ to $t = 18$
- then decelerates at a constant rate from $t = 18$ to $t = 24$, crossing the finishing line with speed $U\,\mathrm{m\,s^{-1}}$

Using the model,

(a) find the acceleration of the athlete during the first 4 s of the race, stating the units of your answer,

(2)

(b) find the distance covered by the athlete during the first 18 s of the race,

(3)

(c) find the value of U.

(3)

a) in the first 4s,

$v = 10$, $u = 0$, $a = ?$, $t = 4$

$v = u + at$ ⟵ because accelerating at a constant rate

$a = \frac{v}{t} = \frac{10\ \mathrm{ms^{-1}}}{4\ \mathrm{s}}$ (1) $= 2.5\ \mathrm{ms^{-2}}$ (1)

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Question 2 continued

b) distance covered = area under the graph

from $t=0$ to $t=4$;

$A = \frac{1}{2} \times 4 \times 10 = 20$ m (1)

from $t=4$ to $t=18$;

$A = 10 \times (18-4) = 140$ m

$\therefore$ Total area = total distance covered = $20 + 140$ (1)

$= 160$ m (1)

c) from $t=18$ to $t=24$,

athlete decelerates at a constant rate. (can use suvat)

$s = 40$
$u = 10$
$v = U$
$a =$
$t = 6$

total race distance

$s = 200$ m $- 160$ m

distance covered from $t=0$ to $t=18$

$= 40$ m (1)

$s = \frac{1}{2}(u+v)t$

$40 = \frac{1}{2}(10+U)6$ (1)

$\frac{40 \times 2}{6} = 10 + U$

$13\frac{1}{3} - 10 = U$

$U = 3\frac{1}{3}$ (1)

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3.

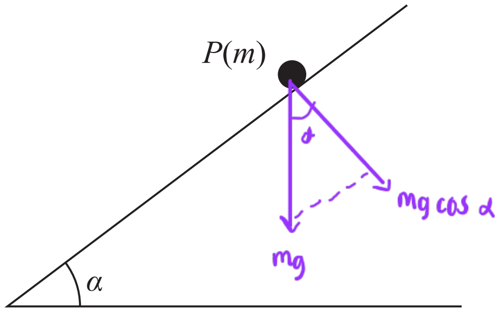


Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan\alpha = \dfrac{5}{12}$
- the coefficient of friction between P and the plane is μ, where $\mu < \dfrac{5}{12}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P,

(2)

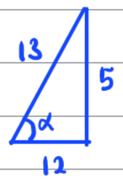
(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{13}g(5 - 12\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu \geqslant \dfrac{5}{12}$

(1)



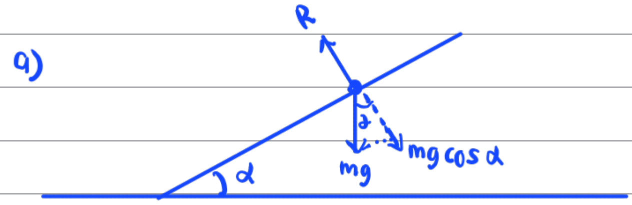
$\tan\alpha = \dfrac{5}{12}$

$\sin\alpha = \dfrac{5}{13}$

$\cos\alpha = \dfrac{12}{13}$

Question 3 continued

a)

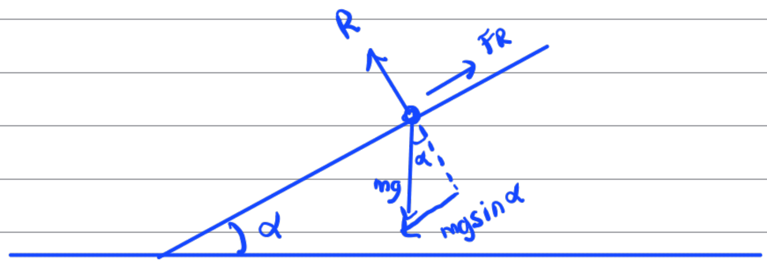


Resultant (↖) :

$R = mg\cos\alpha$ (1)

$R = \frac{12}{13}mg$ (1)

b)



Resultant (↙) : where F_R : frictional force $= \mu R$ (1)

$mg\sin\alpha - F_R = ma$ (1)

$mg\left(\frac{5}{13}\right) - \mu R = ma$

$\cancel{m}g\left(\frac{5}{13}\right) - \mu\left(\frac{12}{13}\cancel{m}g\right) = \cancel{m}a$

$a = \frac{1}{13}g(5 - 12\mu)$ (1)

c) if substitute $\mu = \frac{5}{12}$ into equation of a :

$a = \frac{1}{13}g\left(5 - 12\left(\frac{5}{12}\right)\right)$, $a = 0$. Hence, P would not move if $\mu \geqslant \frac{5}{12}$.
(1)

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

[*In this question,* **i** *is a unit vector due east and* **j** *is a unit vector due north. Position vectors are given relative to a fixed origin O.*]

At time t seconds, $t \geqslant 1$, the position vector of a particle P is $\mathbf{r}$ metres, where

$$\mathbf{r} = ct^{\frac{1}{2}}\mathbf{i} - \frac{3}{8}t^2\mathbf{j}$$

and c is a constant.

When $t = 4$, the bearing of P from O is 135°

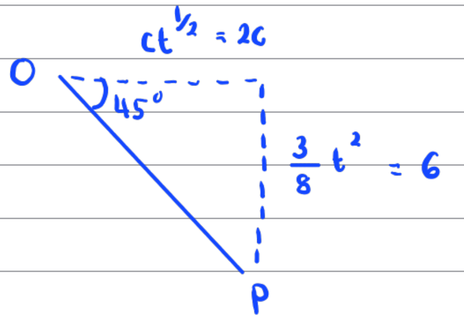
(a) Show that $c = 3$ **(3)**

(b) Find the speed of P when $t = 4$ **(4)**

When $t = T$, P is accelerating in the direction of $(-\mathbf{i} - 27\mathbf{j})$.

(c) Find the value of T. **(4)**

a)



$\Rightarrow \; 2c\underline{i} - 6\underline{j}$ 1

when $t = 4$, $\quad c(4)^{1/2} = 2c$

$\frac{3}{8}(4)^2 = 6$

using trigonometry : $\tan 45^\circ = \frac{6}{2c}$ 1

$2c = 6$

$c = 3$ (shown) 1

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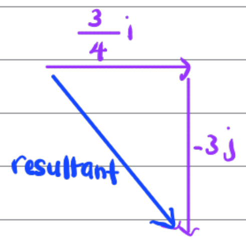
Question 4 continued

b) $r = 3t^{1/2}\underline{i} - \frac{3}{8}t^2\underline{j}$

speed, $v = \frac{dr}{dt} = \frac{3}{2}t^{-1/2}\underline{i} - \frac{3}{4}t\underline{j}$ (1) (1)

when $t = 4$, $\frac{3}{2}(4)^{-1/2}\underline{i} - \frac{3}{4}(4)\underline{j}$

$= \frac{3}{4}\underline{i} - 3\underline{j}$ (1)



resultant $= \sqrt{(3/4)^2 + (-3)^2}$

$= \frac{\sqrt{153}}{4}$ (1)

c) acceleration, $a = \frac{dv}{dt}$ (1)

$= -\frac{3}{4}t^{-3/2}\underline{i} - \frac{3}{4}\underline{j}$ (1)

when $t = T$, acceleration of P $= (-\underline{i} - 27\underline{j})$

$\frac{-\frac{3}{4}T^{-3/2}}{-\frac{3}{4}} = \frac{-1}{-27}$ (1)

$T^{-3/2} = \frac{1}{27}$

$\frac{1}{T^{3/2}} = \frac{1}{27}$

$T^{3/2} = 27$

$T = 9$ (1)

(Total for Question 4 is 11 marks)

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5.

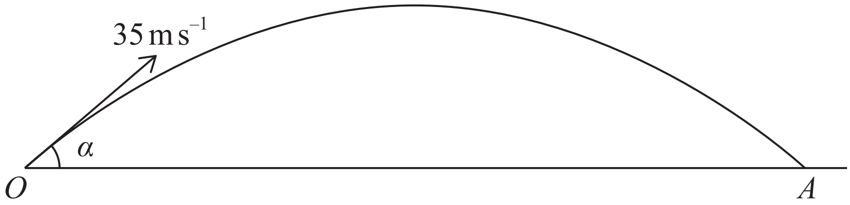

Figure 4

At time $t = 0$, a small stone is projected with velocity $35\,\text{m}\,\text{s}^{-1}$ from a point O on horizontal ground.

The stone is projected at an angle α to the horizontal, where $\tan\alpha = \dfrac{3}{4}$

In an initial model

- the stone is modelled as a particle P moving freely under gravity
- the stone hits the ground at the point A

Figure 4 shows the path of P from O to A.

For the motion of P from O to A

- at time t seconds, the horizontal distance of P from O is x metres
- at time t seconds, the vertical distance of P above the ground is y metres

(a) Using the model, show that

$$y = \frac{3}{4}x - \frac{1}{160}x^2$$

(6)

(b) Use the answer to (a), or otherwise, to find the length OA.

(2)

Using the model, the greatest height of the stone above the ground is found to be H metres.

(c) Use the answer to (a), or otherwise, to find the value of H.

(2)

- The model is refined to include air resistance.

Using this new model, the greatest height of the stone above the ground is found to be K metres.

(d) State which is greater, H or K, justifying your answer.

(1)

(e) State one limitation of this refined model.

(1)

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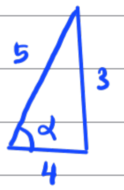
Question 5 continued



$\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

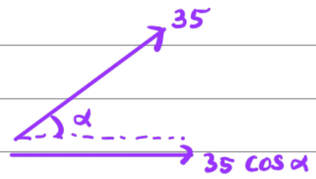
a) At $t = 0$,

Solving horizontally : $s = x$

$u = 35\cos\alpha$

$t = t$

$a = 0$



$s = ut \Rightarrow x = 35\cos\alpha\,(t)$ (1)

$= 35\left(\frac{4}{5}\right)(t)$

$x = 28t$

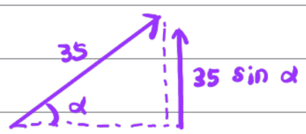
$t = \frac{x}{28}$ — (1) (1)

Solving vertically : $s = y$

$u = 35\sin\alpha$

$t = t$

$a = -g$



(1)

$s = ut + \frac{1}{2}at^2 \Rightarrow y = 35\sin\alpha\,(t) + \frac{1}{2}(-g)(t)^2$

$= 35\left(\frac{3}{5}\right)(t) - \frac{1}{2}gt^2$

$y = 21t - \frac{1}{2}gt^2$ — (2)

(1)

substitute (1) into (2) to eliminate t

$y = 21\left(\frac{x}{28}\right) - \frac{1}{2}g\left(\frac{x}{28}\right)^2$ (1)

$y = \frac{3}{4}x - \frac{1}{160}x^2$ (1)

Question 5 continued

b) O and A is when $y = 0$.

$$0 = \frac{3}{4}x - \frac{1}{160}x^2 \quad ①$$

$$x = \frac{-\frac{3}{4} \pm \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(-\frac{1}{160}\right)(0)}}{2\left(-\frac{1}{160}\right)}$$

$$= \left(-\frac{3}{4} \pm \frac{3}{4}\right)(-80)$$

$x = 0$ or $x = \left(\frac{-6}{4}\right)(-80)$

$x = 120$

$\therefore$ $x = 0$ is at O.

$\therefore$ $x = 120$ is at A.

$\therefore$ OA is 120 m. ①

c) The stone is the highest when $\frac{dy}{dx} = 0$.

$$y = \frac{3}{4}x - \frac{1}{160}x^2$$

$$\frac{dy}{dx} = \frac{3}{4} - \frac{1}{80}x$$

$$\frac{3}{4} - \frac{1}{80}x = 0$$

greatest height, H is at $x = 60$ m

$$x = \frac{3}{4}(80) = 60 \text{ m} \quad ①$$

$$\therefore \; y = \frac{3}{4}(60) - \frac{1}{160}(60)^2$$

$= 45 - 22.5 = 22.5$. H is 22.5 m. ①

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Question 5 continued

d) H would be greater than K as the air resistance would slow down the stone. 1

e) The size of the stone is not taken into account. 1

(Total for Question 5 is 12 marks)

6.

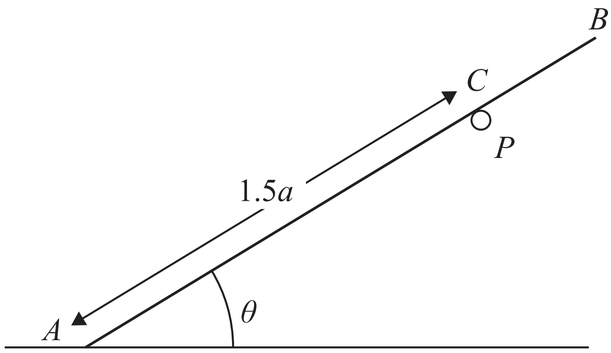


Figure 5

Figure 5 shows a uniform rod AB of mass M and length $2a$.

- the rod has its end A on rough horizontal ground
- the rod rests in equilibrium against a small smooth fixed horizontal peg P
- the point C on the rod, where $AC = 1.5a$, is the point of contact between the rod and the peg
- the rod is at an angle θ to the ground, where $\tan\theta = \dfrac{4}{3}$

The rod lies in a vertical plane perpendicular to the peg.

The magnitude of the normal reaction of the peg on the rod at C is S.

(a) Show that $S = \dfrac{2}{5}Mg$

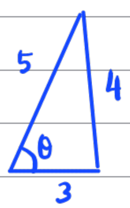
(3)

The coefficient of friction between the rod and the ground is μ.

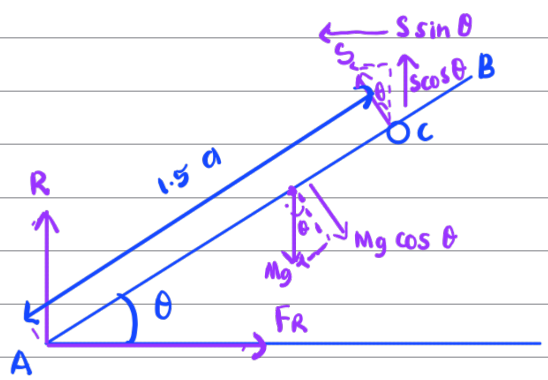
Given that the rod is in limiting equilibrium,

(b) find the value of μ.

(6)



$\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$



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Question 6 continued

a) By taking moments about A : (1)

$S \times 1.5a = Mg\cos\theta \times a$ (1)

$\frac{3}{2}S\cancel{a} = \frac{3}{5}Mg\cancel{a}$

$S = \frac{2}{3} \times \frac{3}{5}Mg$

$S = \frac{2}{5}Mg$ (shown) (1)

b) when rod is at limiting equilibrium, $F_R = \mu R$. (1)

Resolve horizontally : (1)

$F_R = S\sin\theta$

$F_R = \frac{4}{5}S$ (1)

Resolve vertically : (1)

$R + S\cos\theta = Mg$

$R = Mg - \frac{3}{5}S$ (1)

$\therefore$ since $F_R = \mu R$

$$\mu = \frac{F_R}{R} = \frac{\frac{4}{5}S}{Mg - \frac{3}{5}S} = \frac{\frac{4}{5}\left(\frac{2}{5}Mg\right)}{Mg - \frac{3}{5}\left(\frac{2}{5}Mg\right)}$$

$$= \frac{\frac{8}{25}Mg}{\frac{19}{25}Mg} = \frac{8}{19} = 0.421$$ (1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

TOTAL FOR MECHANICS IS 50 MARKS